

Precise Molality and Utilisation with Real Gases

David W Gold

Torquay, Devon, England UK

Email: david.gold@scienceau.uk

Updated: 15 August 2020

Abstract

The objective of this paper is twofold. First, in extending the current thermodynamic properties of mass, temperature, volume and pressure to include the substantiation of mass density as a thermodynamic property, it will be shown that mass density is not just a straightforward ratio of mass and volume, but a representation of a true ratio when heterogeneous states of matter are in an equilibrium state. It will also be shown that, for a given gas of constant mass, the product of density and temperature remain constant irrespective of change in volume and/or pressure. Second, in this paper, where molality is used as a generic term relating to the unitary mole, provides a methodological approach in establishing precise molality by a refinement of the Avogadro principle and, consequently, dispenses with the gas constant. Computation is then specific when dealing with real gases.

PACS numbers: 01.40.-d, 01.55.+b, 51.30.+i

1. Introduction

The subject of gases is extensive and research into it is attributed to many great contributors of the past. This paper deals only with new aspects of the subject, in particular the inherent imprecisions in the computability of gases. In relation to this, fundamental rectification is long overdue and paramount for future education on the subject.

The Boltzmann constant, named after Ludwig Boltzmann, is a physical constant statistically relating the mean kinetic energy of particles in gases with the temperature of gases [1]. It results in the universal use of the gas constant, which being only appropriate for an ideal gas, led to the development of the ideal gas law in 1834 as an approximation in the computation of gases.

Named after Amedeo Avogadro, Avogadro's principle of 1811, or sometimes referred to as Avogadro's law, states: *equal volumes of all gases, at the same temperature and pressure, have the same number of molecules* [2]. It will be demonstrated that the stated term *equal volumes* is imprecise when comparing different gases, and rectification of this term will further remove inconsistencies in the computation of gases.

2. The properties of gases

The properties of mass and volume are dependent on the amount of a substance and are classified as *extensive* properties. In contrast, the properties of mass density, temperature and pressure are independent of the amount of a substance and are classified as *intensive* properties.

For example, if a quantity of gas is divided into sub-portions where the different masses and volumes of each sub-portion, retain proportionality to the original, then the outcome of mass density, temperature and any pressure within the gas remain unaltered in each sub-portion.

In considering that the intensive property of mass density is a ratio of the extensive properties, each property can be ordered by dominancy; based on the priority of their interdependency on each other. The order is mass, temperature, volume, pressure and mass density. The permutation of interdependencies between intensive properties will be highlighted during the course of this paper.

2.1 Pressure

It is important to distinguish between two fundamental forms of pressure: accumulative and thermal pressure. Accumulative pressure, a contractive pressure, is the result of gravitational influence. It is consequential to atmospheric pressure and this influence increasingly accumulates within matter. *Mass density is dependent on accumulative pressure.* Thermal pressure however, is purely attributable to thermodynamics and is a repulsive pressure which is, as shown later, non-contributory to mass matter.

Axiom 1: Thermal pressure modulates an unrestricted volume by being in equipoise to accumulative pressure, and is only discernible when the volume is restricted.

2.2 Intermolecular forces

Deriving the gas equation from the kinetic theory is based on a fundamental assumption that the molecules exert no forces on each other. It is known however, that large intermolecular forces of attraction exist in liquids and solids (cohesive forces) and therefore, the same forces must exist in the gaseous states, but are much weaker in strength due to the greater average distance between molecules [3]. By this inference, intermolecular forces decrease in strength with increasing molecular separation. Molecular kinetic motion/energy is dependent on temperature. Also, *pressure is dependent on temperature.* At first, there would appear to be a correlation between molecular kinetic motion/energy and pressure, and this is the conclusion of the kinetic theory: the notion that pressure results from the accumulative kinetic energy of atoms or molecules striking a container's surfaces [4].

It is forwarded from an alternative reasoning that thermal pressure result from *static* intermolecular repulsive forces, whereby strength is unaffected by temperature and, by the same reasoning, the strength of intermolecular attractive/cohesive forces is affected and is therefore dependent upon temperature. The strength of this *dynamic* force will be inversely proportional to molecular kinetic motion/energy. Equivalence in interactions between these two opposing forces will produce an equilibrium state within a closed system, equating to symmetry between the two forces, whereby any loss of equilibrium will correspond to a differential in force strengths arising from spontaneous symmetry-breaking. Asymmetry in these force strengths will be the cause of expansion or contraction in matter until an equilibrium state is re-established, and any restricted asymmetry in the gaseous states will be the cause of thermal pressure.

2.3 Mass, volume and density

The major components of mass are volume and density. The properties of density ρ , volume V and thermal pressure p of a gas are considered in the following equation, where mass m and atmospheric pressure p_a remain constant:

$$\rho = \frac{m p_a}{V p} \quad (2.31)$$

Eqn. 2.31 forms Proposition 1. In addition to the well-understood changes in pressure and volume, variability in mass density is accounted for in the following theorem:

Theorem 1: *In relation to atmospheric pressure, the mass density of a given gas is proportional to the mass of the gas and inversely proportional to the product of volume and thermal pressure.*

Lemma 1: In the case of an unsuppressed volume of gas, a change in temperature will cause a change in volume, but thermal pressure remains constant with atmospheric pressure, in conformity with Charles' law of 1787 [5]. The resultant change in the mass density of Eqn. 2.31 is consistent with Theorem 1.

Lemma 2: In the case of a fixed volume of gas, a change in temperature will cause a change in thermal pressure, but volume will remain constant, in conformity with Gay–Lussac's (Amontons') law of 1808 [6–7]. The resultant change in the mass density of Eqn. 2.31 is consistent with Theorem 1.

Lemma 3: In the case of a compressed volume of gas, where the initial constant temperature is re-obtained, this will cause a change in thermal pressure and volume, in conformity with Boyle–Mariotte's law of 1654 [8]. The resultant mass density of Eqn. 2.31 remains constant and is consistent with Theorem 1.

	Mass of Gas g	Initial Temperature K	<i>Specimen</i> Temperature K	Initial Volume L	Resultant/ <i>Specimen</i> Volume L	Resultant Pressure atm	Initial Mass Density g/L	Resultant Mass Density g/L	Constant $\rho' T'$
Lemma 1: Unsuppressed	83.8	273	173	22.347	14.161	1	3.75	5.918	1023.750
Lemma 2: Fixed	83.8	273	173	22.347	22.347	0.634	3.75	5.918	1023.750
Lemma 3: Compressed	83.8	273	273	22.347	14	1.596	3.75	3.750	1023.750

Table 1: Mass density of a gas ¹

From the above lemmas, it can be seen from the deduction that change in density only occurs when there is a change in temperature. Therefore, *mass density is dependent on temperature*. The deduction establishes that for a given gas, of constant mass, the product of density and temperature remains constant:

$$\rho_1 T_1 = \rho_2 T_2 \quad (2.32)$$

Eqn. 2.32 forms Proposition 2, which is affirmed by the corresponding coefficients:

$$\frac{1}{V_i} \left(\frac{\partial V}{\partial T} \right)_p = \frac{1}{p_i} \left(\frac{\partial p}{\partial T} \right)_V = - \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_m \quad (2.33)$$

Lemma 1 Lemma 2 Lemmas 1 & 2

The validity of propositions 1 and 2 will later be substantiated.

¹ See criterion in 3 i.e. $V_i = m/\rho_i$. Calculated results are displayed rounded to three decimal places.

3. Precise molality

The unitarity mole (which comes from a reference to molecules), as derived by Avogadro, is a comparative or relative unit applicable to all matter states. This relative aspect will be represented by the upper-case R used in the notation of variables that pertain to the mole. These variables are then distinguishable concerning content.

A criterion exists whereby the achievement of greater accuracy in computation entails the use of a higher degree of precision.

3.1 Atomic mass units

Atomic mass units are based on the average naturally occurring isotopes found in each atomic element. The masses used to compute the molecular mass are found in a table of isotopic masses [9] and are not found in a typical periodic table. As a convention, for each element and substance, the notations used are u = atomic mass units and N = molecular element number, which is applicable to elements only: the number of atoms combined to form a molecular element, else in all other instances $N = 1$. For substances, the atomic mass units are obtained as illustrated:

Substance		Protons	Neutrons	Nucleon Number	Initial Atomic Mass Units u	Resultant Atomic Mass Units u
Dimethyl ether CH_3OCH_3	C	6	6	12	12.010736	$\times 1$ 12.010736
	H_3	3	0	3	1.007941	$\times 3$ 3.023823
	O	8	8	16	15.999400	$\times 1$ 15.999400
	C	6	6	12	12.010736	$\times 1$ 12.010736
	H_3	3	0	3	1.007941	$\times 3$ 3.023823
Dimethyl ether:		26	20	46		46.068518

Illustration 1: Atomic mass units of a substance

3.2 Relative aspects of matter

In establishing the following relative molar components, units of grams, litres and, for pressure, atm are universally applied. Constant mass of an individual element or substance is uniformly expressed by its molar mass:

$$R_m = uNm_u N_A \quad \text{g mol}^{-1} \quad (3.21)$$

where m_u = atomic mass constant (in grams) and N_A = Avogadro constant. Molar volume is proportional to molar mass and is inversely proportional to mass density:¹

$$R_v = \frac{R_m}{\rho} \quad \text{L mol}^{-1} \quad (3.22)$$

where ρ = mass density. Precise molality is demonstrated by the slight variance in molar volume for each gas and, consequentially, establishes a crucial difference between gases. Relative

¹ Different temperatures and atmospheric pressures affect mass density and, thereby, although not essential, it is consistent for the molar components of different elements and substances to be initialised at standard temperature and pressure (STP) i.e. 273.15 K & 1 atm. In doing so, will emphasise their comparative relative aspects.

equivalence between the different mass densities of individual elements and substances is achieved by conversion to a ‘specific density’:¹

$$R_\rho = \frac{\rho}{uN} \quad \text{g L}^{-1} \text{u}^{-1} \quad (3.23a)$$

$$\rho = uNR_\rho \quad (3.23b)$$

$$uNR_\rho = \frac{uNm_uN_A}{R_v} \quad (3.23c)$$

As $m_uN_A = 1$ gram, specific density is thereby reciprocal to molar volume:

$$R_\rho = \frac{1}{R_v} \quad (3.23d)$$

A gaseous system is a closed system defined by its volume. Application of the molar unit becomes apparent when the size of a homogeneous system is unknown, whereby it allows evaluating a sub-portion of the system in which the results obtained will be consistent with the whole system. In the case of large heterogeneous systems, it allows evaluating sub-portions of the system, for example at different pressures (atmospheric or thermal).

If the initial volume is known, then the number of moles of a system is derived in a straightforward way from the initial volume being inversely proportional to molar volume:

$$n = \frac{V_i}{R_v} \quad (3.24)$$

A specific gas constant relevant for a given gas is proportional to the product of molar volume and atmospheric pressure (in any specified units) and is inversely proportional to the initialising temperature:

$$R_s = \frac{R_v p_a}{T} \quad (3.25)$$

The specific gas constant, when substituted into the ideal gas law, converts it to a specific gas law and, from its formulation, is reconcilable with molar volume:

$$R_s T' = R_v (T'/T) p_a \quad (3.26a)$$

Thereby, this will give rise to a new equation of state for gases without the need for a gas constant:

$$pV = nR_v (T'/T) p_a \quad (3.26b)$$

Law 1: In relation to atmospheric pressure, the product of thermal pressure and volume of a given gas is directly proportional to its molar volume, which is proportional to any change in absolute temperature.

The above law leads to fully correlating the individual properties of any given gas:

$$\frac{pV}{p_a} = \frac{nR_m}{\rho} = \frac{nR_v T'}{T} \quad (3.27)$$

¹ A dimensionless unit, u, is assigned to atomic mass units in order to notate a distinction between mass density and specific density.

Elements (gases only)	Element: Chemical Formula	Protons (Element: Atomic Number)		Nucleon Number	Atomic Mass Units u	Molecular Element Number N	Mass Density ρ (g L^{-1} or kg m^{-3})	Molar Mass R_m (g mol^{-1})	Molar Volume R_v (L mol^{-1})	Specific Density R_p ($\text{g L}^{-1} \text{u}^{-1}$)	Specific Gas Constant R_s ($\text{L atm K}^{-1} \text{mol}^{-1}$)
		Neutrons									
Hydrogen	H ₂	1	0	1	1.007941	2	0.0899	2.015882	22.423604	0.044596	0.082093
Helium	He	2	2	4	4.002602	1	0.1785	4.002602	22.423541	0.044596	0.082092
Nitrogen	N ₂	7	7	14	14.006743	2	1.251	28.013486	22.392875	0.044657	0.081980
Oxygen	O ₂	8	8	16	15.999405	2	1.429	31.998810	22.392449	0.044658	0.081979
Fluorine	F ₂	9	10	19	18.998403	2	1.696	37.996806	22.403777	0.044635	0.082020
Neon	Ne	10	10	20	20.180046	1	0.900	20.180046	22.422273	0.044599	0.082088
Chlorine	Cl ₂	17	18	35	35.452539	2	3.214	70.905078	22.061319	0.045328	0.080766
Argon	Ar	18	22	40	39.947677	1	1.784	39.947677	22.392196	0.044658	0.081978
Krypton	Kr	36	48	84	83.799325	1	3.75	83.799325	22.346487	0.044750	0.081810
Xenon	Xe	54	77	131	131.292481	1	5.9	131.292481	22.252963	0.044938	0.081468
Radon	Rn	86	136	222	222.017570	1	9.73	222.017570	22.817839	0.043825	0.083536
Substances (sample gases)											
Ozone	O ₃	8	8	16	15.999405	3	2.144	47.998215	22.387227	0.044668	0.081959
Methane	CH ₄	10	6	16	16.042500	1	0.716	16.042500	22.405726	0.044631	0.082027
Ammonia	NH ₃	10	7	17	17.030566	1	0.77	17.030566	22.117618	0.045213	0.080972
Steam (373.15 K)	H ₂ O	10	8	18	18.015287	1	0.60	18.015287	30.025478	0.033305	0.080465
Carbon dioxide	CO ₂	22	22	44	44.009546	1	1.98	44.009546	22.227043	0.044990	0.081373

Table 2: Relative molar components of gases at STP

Table A in the appendix contains a list of relative molar component quantities for each element and sample substances at STP or where temperatures for non-gases are specified. It can be deduced from the information in this table that $R_v < 1$ for all liquids and solids, and $R_v > 1$ for all gases. Furthermore, there will be a divergent state when $R_v = 1$ as, within the bounds of a critical temperature, momentary occurring at the approximate mean point during a phase transition involving vaporisation/condensation or sublimation/deposition. These R_v conformities are applicable for all individual elements and substances, regardless of pressure.

This deduction concludes in Rule 1: *At or below a critical temperature and regardless of pressure, the approximate mean point during a phase transition between a liquid or sublimation of a solid to a gas or vice versa of an individual element or substance (or in their summing) occurs at the point when their molar volume traverses the unitarity volume.*

It should be noted that a specific gas or gaseous mixture above its critical temperature will remain in a gaseous state regardless of a continuous increase in pressure and irrespective of if the molar volume has traversed below the unitarity volume. An example of critical temperature is inside the Sun where, descending at the Sun's tachocline transition region, the molar volume of the gas plasma is less than one, but the internal temperature exceeds the critical temperature where, conversely, the gas plasma would transform into a liquid state.

Importantly, when molar volume traverses the unitarity volume the atoms and molecules in elements and substances are at the juncture where, in liquids and solids, large intermolecular attractive/cohesive forces occur to where the kinetic theory fundamentally assumes, in the gaseous state, atoms and molecules exert no forces on each other. It is implausible that these forces would fundamentally break down or form at this juncture. As alternatively reasoned, these forces would persist at strengths relational to their molecular separation whereby the persistence of these forces must then cause a bona fide non-trivial effect in gases.

3.3 Utilisation with real gases

Earth's Atmospheric Gases	Atomic Mass Units u	Molecular Element Number N	Mass Density ρ (g L^{-1} or kg m^{-3}) @ STP	Molar	Molar	% of Earth's Atmospheric Gases by Volume	Fractional Molar Volume by %	Fractional Mole by Volume	Fractional Molar Mass	Fractional Specific Density	Dalton's partial pressures @ 450 K (atm mol^{-1})	
				Mass R_m (g mol^{-1})	Volume R_v (L mol^{-1})							
Nitrogen	N_2	14.00674	2	1.251	28.013	22.39287	78.0825	17.4849162	0.780830392	21.8737813	0.0348696	1.2441053
Oxygen	O_2	15.99941	2	1.429	31.999	22.39245	20.944	4.6898746	0.209437469	6.7017498	0.0093530	0.3811724
Argon	Ar	39.94768	1	1.784	39.948	22.3922	0.932	0.2086953	0.009319782	0.3723036	0.0004162	0.0211753
Carbon dioxide	CO_2	44.00955	1	1.98	44.01	22.22704	0.039	0.0086685	0.000387115	0.0170367	0.0000174	0.0009690
Neon	Ne	20.18005	1	0.900	20.18	22.42227	0.001818	0.0004076	0.000018204	0.0003674	0.0000008	0.0000209
Helium	He	4.002602	1	0.1785	4.0026	22.42354	0.000524	0.0001175	0.000005247	0.0000210	0.0000002	0.0000012
Methane	CH_4	16.0425	1	0.716	16.043	22.40573	0.000179	0.0000401	0.000001791	0.0000401	0.0000001	0.0000023
Key: Atomic Element, Molecular Substance				Sum Total:		100	22.392720	1	28.965300	0.0446574	1.64745	atm
Source: Wikipedia - Atmosphere of Earth				Specific Gas Constant:		0.0819796	Mass Density:		1.2935144			

Table 3: Composition of Earth's atmosphere (dry air)

In using the earth's atmosphere as an example, it can be demonstrated that any gaseous mixture can be treated as a specific gas when the cumulative molar components are established by summing the component quantities for each gaseous atomic element and molecular substance. If the proportions are by percentage of mass, then the volume and mass aspects are interchanged. For dry air at the earth's surface where:

$$\begin{aligned} \text{Atmospheric Pressure: } & p_a = 1 \quad \text{atm} \\ \text{Temperature: } & T = 273.15 \quad \text{K} \end{aligned}$$

the relative molar components are:

$$\begin{aligned} \text{Molar mass: } & R_m = 28.965300 \quad \text{g mol}^{-1} \\ \text{Molar volume: } & R_v = 22.392720 \quad \text{L mol}^{-1} \\ \text{Specific density: } & R_\rho = 0.0446574 \quad \text{g L}^{-1} \text{ u}^{-1} \\ \text{Specific gas constant: } & R_s = 0.0819796 \quad \text{L atm K}^{-1} \text{ mol}^{-1} \\ \text{Mass density: } & \rho = 1.2935144 \quad \text{g/L or kg/m}^3 \end{aligned}$$

In establishing the values for the molar components of a gaseous mixture, aspects like pressure are calculated in a straightforward way without the need to resort to the application of Dalton's 1793 law of partial pressures [10]. In the example produced in Table 3, the concluding thermal pressure p established by applying Dalton's partial pressures within a fixed volume of 1 mole and at 450 K can also be obtained by use of the specific gas constant:

$$\frac{R_s T'}{R_v} = p \quad (3.31)$$

$$(0.0819796 \text{ L atm K}^{-1} \text{ mol}^{-1} \times 450 \text{ K}) / 22.39272 \text{ L mol}^{-1} = 1.64745 \text{ atm}$$

Alternatively this can be obtained in the application of the new equation of state for real gases, as defined by Law 1, where in this instance of fixed volume, $V = nR_v$ and therefore will cancel out, leaving the differential in temperature that ascertains the change in thermal pressure, as supported by Gay-Lussac's (Amontons') Law [6–7]:

$$\frac{nR_v T' p_a}{V T} = p \quad (3.32)$$

$$(450 \text{ K} / 273.15 \text{ K}) \times 1 \text{ atm} = 1.64745 \text{ atm}$$

3.3.1 Critical points of gases

Element (gases)	Data						Proposition 1			Proposition 2		
	Atomic Mass Units u	Molecular Element Number N	Mass Density ρ @ T (g L^{-3})	Critical Pressure p_c (atm)	Critical Temperature T_c K	Molar Mass R_m (g mol^{-1})	Molar Volume R'_v (L mol^{-1})	Mass Density ρ @ T_c (g L^{-3})	Reaffirms Critical Pressure p_c (atm)	Constant $\rho T = \rho_c T_c$		
Hydrogen	H ₂	1.00794	2	0.0899	12.80	33.2	2.01588	0.21293	0.73964	12.80	24.5562	24.5562
Helium	He	4.00260	1	0.1785	2.24	5.19	4.0026	0.19021	9.39447	2.24	48.7573	48.7573
Nitrogen	N ₂	14.00670	2	1.251	33.50	126.2	28.0134	0.30883	2.70769	33.50	341.711	341.711
Oxygen	O ₂	15.99940	2	1.429	49.80	154.6	31.9988	0.25450	2.52478	49.80	390.331	390.331
Fluorine	F ₂	18.99840	2	1.696	51.50	144.3	37.9968	0.22982	3.21041	51.50	463.262	463.262
Neon	Ne	20.17970	1	0.900	27.20	44.4	20.1797	0.13399	5.53682	27.20	245.835	245.835
Chlorine	Cl ₂	35.45270	2	3.214	76.00	416.9	70.9054	0.44305	2.10579	76.00	877.904	877.904
Argon	Ar	39.94800	1	1.784	48.10	150.8	39.948	0.25701	3.23143	48.10	487.3	487.3
Krypton	Kr	83.80000	1	3.75	54.30	209.3	83.8	0.31534	4.89399	54.30	1024.31	1024.31
Xenon	Xe	131.29000	1	5.9	57.60	289.8	131.29	0.40988	5.56102	57.60	1611.59	1611.59
Radon	Rn	222.00000	1	9.73	61.98	377	222	0.50809	7.04973	61.98	2657.75	2657.75
Substance (sample gases)												
Methane	CH ₄	16.04276	1	0.716	45.79	190.8	16.04276	0.34180	1.02503	45.79	195.575	195.575
Ammonia	NH ₃	17.03052	1	0.77	111.30	405.5	17.03052	0.29501	0.51868	111.30	210.326	210.326
Steam (373.15 K)	H ₂ O	18.01530	1	0.60	217.70	647.096	18.0153	0.23918	0.34599	217.70	223.89	223.89
Carbon dioxide	CO ₂	44.00980	1	1.98	72.80	304.19	44.0098	0.34001	1.77796	72.80	540.837	540.837

Table 4: Critical points of gases

Data for the critical points of gases is used to further substantiate Proposition 1 for Theorem 1: *In relation to atmospheric pressure, the mass density of a given gas is proportional to the mass of the gas and inversely proportional to the product of volume and thermal pressure.*

Constant mass of a given gas is as expressed by Eqn. 3.21. Molar volume, initially established at STP in table 2 where $T = 273.15$ K (Steam: $T = 373.15$ K) and $p_a = 1$ atm, is updated to include critical temperature and critical pressure:

$$R'_v = R_v[(T_c/T)(p_a/p_c)] \quad (3.33)$$

From Proposition 1:

$$\rho = \frac{R_m p_a}{R'_v p_c} \quad (3.35)$$

Equivalent mass density is derived from specific density, updated from table 2, using critical temperature only:

$$\rho = uNR_\rho(T/T_c) \quad (3.36)$$

A rearrangement of Proposition 1 is $p_c = (R_m p_a)/(R'_v \rho)$. Eqn. 3.36 reaffirms that thermal pressure does not contribute to specific density, and, consequentially, mass density. Eqn. 3.36 also substantiates Proposition 1 and, in addition to the results presented in Table 4, further substantiates Proposition 2, which is that mass density is dependent on temperature.

Table 4 shows that molar volumes has traverse the one litre threshold as declared in Rule 1, where, in the above examples, at a critical transition point, each gas has transformed into a liquid state (in compliance with critical temperatures: $R_v < 1$ for liquids and solids and $R_v > 1$ for gases).

The substantiation of propositions 1 and 2 validates Theorem 1 and is furthered by the fact that Vp/p_a expresses the unsuppressed volume of a gas. The validation of Theorem 1 results in the following classical laws:

Law 2: In relation to atmospheric pressure, the mass density of a given gas remains proportional to the mass and inversely proportional to the unsuppressed volume of the gas, irrespective of change in volume and/or thermal pressure.

Law 3: For a given gas of constant mass, mass density is inversely proportional to change in absolute temperature where the product of density and temperature remain constant.

3.4 Equilibrium state

It has been established that thermal pressure does not contribute to specific density and therefore is an exclusive property of molar volume. Change in molar volume of a specific gas or gaseous mixture is determined from the changes in any or all of its following properties:

$$R'_v = R_v[(T'/T)(p_a/p'_a)(p_a/p)] \quad (3.41)$$

In all heterogeneous states of matter, an equilibrium state is identifiable when the product $R'_v R'_\rho = 1$. Thereby, a non-equilibrium state indicates a distinction in the form of loss of unity between R'_v and R'_ρ , which must correlate with a distinction between the fundamental pressures:

$$p = \frac{p_a}{R'_v R'_\rho} \quad (3.42a)$$

In substituting Eqn. 3.41 for R'_v :

$$R'_\rho = \frac{1}{R_v} \frac{T p'_a}{T' p_a} \quad (3.42b)$$

As established in Eqn. 3.23d, at initialisation specific density is the reciprocal of molar volume:

$$R'_\rho = R_\rho[(T/T')(p'_a/p_a)] \quad (3.42c)$$

Again, is further validation that thermal pressure is non-contributory to specific density and is an exclusive property of molar volume.

4. Conclusions

It has been established that mass density remains consistent with unsuppressed volume, which is a manifestation of an equilibrium state. By deduction, whereas accumulative pressure contributes to mass density, thermal pressure does not affect mass density in any way. This will be a significant paradigm shift in our understanding of mass density. Mass density is not just a straightforward ratio of mass and volume, but it is a representation of a true ratio when substances are in an equilibrium state, and this will be applicable to all heterogeneous states of matter. The implication further emphasises that the alternative reasoning presented in subsection 2.2 is essential to explaining the cause.

From the established proportionality of temperature and mass density, as well as from the alternative reasoning relating to intermolecular forces, it can be further deduced, conjecturally, that, within all matter states, the *static* or consistency of the intermolecular repulsive force will be responsible for determining volume and in equipollence, the *dynamic* intermolecular attractive/cohesive force will be responsible for determining mass density. Furthermore, it has been demonstrated that thermal pressure does not contribute to matter. Therefore, the overall conclusion is that thermal pressure is not a direct consequence of kinetic energy (see lemma 3) but constitutes to the potential energy of a gaseous system in which direct correlations between thermal pressure, potential energy and loss of an equilibrium state must ultimately arise from asymmetry in opposing intermolecular force strengths.

Applied constants

<http://physics.nist.gov/cuu/Constants/index.html>

References/further information

- [1] Feynman, R.P. The Feynman Lectures on Physics Vol 1 (1970) ISBN: 978-0-465-02493-3 Basic Books New York
- [2] Amedeo Avogadro: [Avogadro @ Chemical Heritage Foundation](#)
- [3] Joos, G., Freeman, I.M. Theoretical Physics (1986) ISBN-13: 987-0-486-65227-6 Dover Publications New York 492
- [4] Atkins, P., de Paula, J. Atkins' Physical Chemistry 8th Edition (2006) ISBN: 978-0-19-870072-2 Oxford University Press Oxford 748
- [5] Fullick, P. Physics (1994) ISBN 0-435-57078-1 Heinemann Oxford 141–42
- [6] Barnett, M.K. A brief history of thermometry *J. Chem. Educ.* **18** 8 358 (1941)
[DOI:10.1021/ed018p358](https://doi.org/10.1021/ed018p358)
- [7] Joseph Louis Gay-Lussac: [Gay-Lussac @ Chemical Heritage Foundation](#)
- [8] Robert Boyle: [Boyle @ Chemical Heritage Foundation](#)
- [9] [Table of Isotopic Masses](#)
- [10] John Dalton: [Dalton @ Chemical Heritage Foundation](#)

Appendix

Table A: Relative molar components of elements and substances

Element	Periodic Table Symbol	Protons (Element: Atomic Number)	Neutrons	Nucleon Number	Atomic Mass Units u	Molecular Element Number N	Melting Point (K)	Boiling Point (K)	Mass Density ρ (g L ⁻¹ or kg m ⁻³)	Molar Mass R_m (g mol ⁻¹)	Molar Volume R_v (L mol ⁻¹)	Specific Density R_p (g L ⁻¹ u ⁻¹)	R_p % of Diamond
Hydrogen (H ₂)	H	1	0	1	1.007941	2	13.81	20.28	0.0899	2.01588	22.424	0.045	0.015%
Helium	He	2	2	4	4.002602	1	0.95	4.216	0.1785	4.00260	22.424	0.045	0.015%
Lithium	Li	3	4	7	6.940037	1	453.7	1615	530	6.94004	0.0131	76.368	26%
Beryllium	Be	4	5	9	9.012182	1	1560	3243	1850	9.01218	0.0049	205.278	70%
Boron	B	5	6	11	10.811028	1	2365	4275	2340	10.81103	0.0046	216.446	74%
Carbon	C	6	6	12	12.010736	1	3825	5100	2260	12.01074	0.0053	188.165	64%
Diamond		6	6	12	12.010736	1	3825	5100	3520	12.01074	0.0034	293.071	100%
Graphene		6	6	12	12.010736	1	3825	5100	4120	12.01074	0.0029	343.026	117%
Nitrogen (N ₂)	N	7	7	14	14.006743	2	63.15	77.344	1.251	28.01349	22.393	0.045	0.015%
Oxygen (O ₂)	O	8	8	16	15.999405	2	54.8	90.188	1.429	31.99881	22.392	0.045	0.015%
Fluorine (F ₂)	F	9	10	19	18.998403	2	53.55	85	1.696	37.99681	22.404	0.045	0.015%
Neon	Ne	10	10	20	20.180046	1	24.55	27.1	0.900	20.18005	22.422	0.045	0.015%
Sodium	Na	11	12	23	22.989770	1	371	1156	970	22.98977	0.0237	42.193	14%
Magnesium	Mg	12	12	24	24.305052	1	922	1380	1740	24.30505	0.0140	71.590	24%
Aluminium	Al	13	14	27	26.981538	1	933.5	2740	2700	26.98154	0.0100	100.068	34%
Silicon	Si	14	14	28	28.085413	1	1683	2630	2330	28.08541	0.0121	82.961	28%
Phosphorus (White)	P	15	16	31	30.973762	1	317.3	553	1820	30.97376	0.0170	58.759	20%
Phosphorus (Red)	P	15	16	31	30.973762	1	317.3	553	2200	30.97376	0.0141	71.028	24%
Phosphorus (Black)	P	15	16	31	30.973762	1	317.3	553	2700	30.97376	0.0115	87.171	30%
Sulphur	S	16	16	32	32.066085	1	392.2	717.82	2070	32.06608	0.0155	64.554	22%
Chlorine (Cl ₂)	Cl	17	18	35	35.452539	2	172.2	239.18	3.214	70.90508	22.061	0.045	0.015%
Argon	Ar	18	22	40	39.947677	1	83.95	87.45	1.784	39.94768	22.392	0.045	0.015%
Potassium	K	19	20	39	39.098301	1	336.8	1033	860	39.09830	0.0455	21.996	8%
Calcium	Ca	20	20	40	40.078023	1	1112	1757	1550	40.07802	0.0259	38.675	13%
Scandium	Sc	21	24	45	44.955910	1	1814	3109	2990	44.95591	0.0150	66.510	23%
Titanium	Ti	22	26	48	47.866750	1	1935	3560	4540	47.86675	0.0105	94.847	32%
Vanadium	V	23	28	51	50.941472	1	2163	3650	6110	50.94147	0.0083	119.942	41%
Chromium	Cr	24	28	52	51.996138	1	2130	2945	7190	51.99614	0.0072	138.280	47%
Manganese	Mn	25	30	55	54.938050	1	1518	2235	7440	54.93805	0.0074	135.425	46%
Iron	Fe	26	30	56	55.845150	1	1808	3023	7874	55.84515	0.0071	140.997	48%
Cobalt	Co	27	32	59	58.933200	1	1768	3143	8900	58.93320	0.0066	151.018	52%
Nickel	Ni	28	31	59	58.693356	1	1726	3005	8900	58.69336	0.0066	151.636	52%
Copper	Cu	29	35	64	63.545644	1	1356.6	2840	8960	63.54564	0.0071	141.001	48%
Zinc	Zn	30	35	65	65.395567	1	692.73	1180	7130	65.39557	0.0092	109.029	37%
Gallium	Ga	31	39	70	69.723072	1	302.92	2478	5910	69.72307	0.0118	84.764	29%
Germanium	Ge	32	41	73	72.612759	1	1211.5	3107	5320	72.61276	0.0136	73.265	25%
Arsenic	As	33	42	75	74.921596	1	889	-	5780	74.92160	0.0130	77.147	26%
α -Arsenic	As	33	42	75	74.921596	1	889	-	2000	74.92160	0.0375	26.695	9%
Selenium	Se	34	45	79	78.959389	1	494	958	4790	78.95939	0.0165	60.664	21%
Bromine (Br ₂)	Br	35	45	80	79.903529	2	265.95	331.85	3120	159.80706	0.0512	19.524	7%
Krypton	Kr	36	48	84	83.799325	1	116	120.85	3.75	83.79932	22.346	0.045	0.015%
Rubidium	Rb	37	48	85	85.467664	1	312.63	961	1532	85.46766	0.0558	17.925	6%
Strontium	Sr	38	50	88	87.616646	1	1042	1655	2540	87.61665	0.0345	28.990	10%
Yttrium	Y	39	50	89	88.905848	1	1795	3611	4470	88.90585	0.0199	50.278	17%
Zirconium	Zr	40	51	91	91.223647	1	2128	4682	6510	91.22365	0.0140	71.363	24%
Niobium	Nb	41	52	93	92.906378	1	2742	5015	8570	92.90638	0.0108	92.243	31%
Molybdenum	Mo	42	54	96	95.931292	1	2896	4912	10220	95.93129	0.0094	106.535	36%
Technetium	Tc	43	55	98	97.907216	1	2477	4538	11500	97.90722	0.0085	117.458	40%
Ruthenium	Ru	44	57	101	101.064945	1	2610	4425	12370	101.06494	0.0082	122.397	42%
Rhodium	Rh	45	58	103	102.905504	1	2236	3970	12410	102.90550	0.0083	120.596	41%
Palladium	Pd	46	60	106	106.415328	1	1825	3240	12000	106.41533	0.0089	112.766	38%
Silver	Ag	47	61	108	107.868151	1	1235.08	2436	10500	107.86815	0.0103	97.341	33%
Cadmium	Cd	48	64	112	112.411553	1	594.26	1040	8650	112.41155	0.0130	76.949	26%
Indium	In	49	66	115	114.818086	1	429.78	2350	7310	114.81809	0.0157	63.666	22%
Tin (White)	Sn	50	69	119	118.710111	1	505.12	2876	7310	118.71011	0.0162	61.579	21%
Tin (Grey)	Sn	50	69	119	118.710111	1	505.12	2876	5800	118.71011	0.0205	48.859	17%
Antimony	Sb	51	71	122	121.759788	1	903.91	1860	6690	121.75979	0.0182	54.944	19%
Tellurium	Te	52	76	128	127.603125	1	722.72	1261	6240	127.60312	0.0204	48.902	17%
Iodine	I	53	74	127	126.904468	1	386.7	457.5	4930	126.90447	0.0257	38.848	13%
Xenon	Xe	54	77	131	131.292481	1	161.39	165.1	5.9	131.29248	22.253	0.045	0.015%
Caesium	Cs	55	78	133	132.905447	1	301.54	944	1870	132.90545	0.0711	14.070	5%
Barium	Ba	56	81	137	137.326886	1	1002	2078	3590	137.32689	0.0383	26.142	9%

Element	Periodic Table Symbol	Protons (Element: Atomic Number)		Nucleon Number	Atomic Mass Units μ	Molecular Element Number N	Melting Point (K)	Boiling Point (K)	Mass Density ρ (g L ⁻¹ or kg m ⁻³)	Molar Mass R_m (g mol ⁻¹)	Molar Volume R_v (L mol ⁻¹)	Specific Density R_p (g L ⁻¹ u ⁻¹)	R_p % of Diamond
		Neutrons	Neutrons										
Lanthanum	La	57	82	139	138.905449	1	1191	3737	6150	138.90545	0.0226	44.275	15%
Cerium	Ce	58	82	140	140.115722	1	1071	3715	6770	140.11572	0.0207	48.317	16%
Praseodymium	Pr	59	82	141	140.907648	1	1204	3785	6770	140.90765	0.0208	48.046	16%
Neodymium	Nd	60	84	144	144.236127	1	1294	3347	7010	144.23613	0.0206	48.601	17%
Promethium	Pm	61	84	145	144.912744	1	1315	3273	7220	144.91274	0.0201	49.823	17%
Samarium	Sm	62	88	150	150.366344	1	1347	2067	7520	150.36634	0.0200	50.011	17%
Europium	Eu	63	89	152	151.964336	1	1095	1800	5240	151.96434	0.0290	34.482	12%
Gadolinium	Gd	64	93	157	157.252119	1	1585	3545	7900	157.25212	0.0199	50.238	17%
Terbium	Tb	65	94	159	158.925343	1	1629	3500	8230	158.92534	0.0193	51.785	18%
Dysprosium	Dy	66	97	163	162.497030	1	1685	2840	8550	162.49703	0.0190	52.616	18%
Holmium	Ho	67	98	165	164.930319	1	1747	2968	8800	164.93032	0.0187	53.356	18%
Erbium	Er	68	99	167	167.256301	1	1802	3140	9070	167.25630	0.0184	54.228	19%
Thulium	Tm	69	100	169	168.934211	1	1818	2223	9320	168.93421	0.0181	55.169	19%
Ytterbium	Yb	70	103	173	173.037692	1	1092	1469	6970	173.03769	0.0248	40.280	14%
Lutetium	Lu	71	104	175	174.966718	1	1936	3668	9840	174.96672	0.0178	56.239	19%
Hafnium	Hf	72	106	178	178.484971	1	2504	4875	13310	178.48497	0.0134	74.572	25%
Tantalum	Ta	73	108	181	180.947876	1	3293	5730	16650	180.94788	0.0109	92.015	31%
Tungsten	W	74	110	184	183.841779	1	3695	5825	19300	183.84178	0.0095	104.982	36%
Rhenium	Re	75	111	186	186.206706	1	3455	5870	21000	186.20671	0.0089	112.778	38%
Osmium	Os	76	114	190	190.224861	1	3300	5300	22600	190.22486	0.0084	118.807	41%
Iridium	Ir	77	115	192	192.216054	1	2720	4700	22600	192.21605	0.0085	117.576	40%
Platinum	Pt	78	117	195	195.077791	1	2042.1	4100	21450	195.07779	0.0091	109.956	38%
Gold	Au	79	118	197	196.966552	1	1337.58	3130	19300	196.96655	0.0102	97.986	33%
Mercury	Hg	80	121	201	200.599149	1	284.31	629.88	13550	200.59915	0.0148	67.548	23%
Thallium	Tl	81	123	204	204.383317	1	577	1746	11850	204.38332	0.0172	57.979	20%
Lead	Pb	82	125	207	207.216892	1	600.65	2023	11350	207.21689	0.0183	54.774	19%
Bismuth	Bi	83	126	209	208.980383	1	544.59	1837	9750	208.98038	0.0214	46.655	16%
Polonium	Po	84	125	209	208.982416	1	527	1237	9300	208.98242	0.0225	44.501	15%
Astatine	At	85	125	210	209.987131	1	575	610	7000	209.98713	0.0300	33.335	11%
Radon	Rn	86	136	222	222.017570	1	202	211.4	9.73	222.01757	22.818	0.044	0.015%
Francium	Fr	87	136	223	223.019731	1	300	950	1870	223.01973	0.1193	8.385	3%
Radium	Ra	88	138	226	226.025403	1	973	1413	5000	226.02540	0.0452	22.121	8%
Actinium	Ac	89	138	227	227.027747	1	1324	3470	10070	227.02775	0.0225	44.356	15%
Thorium	Th	90	142	232	232.038050	1	2028	5060	11720	232.03805	0.0198	50.509	17%
Protactinium	Pa	91	140	231	231.035879	1	1845	4300	15400	231.03588	0.0150	66.656	23%
Uranium	U	92	146	238	238.028913	1	1408	4407	18950	238.02891	0.0126	79.612	27%
Neptunium	Np	93	144	237	237.048167	1	912	4175	20200	237.04817	0.0117	85.215	29%
Plutonium	Pu	94	150	244	244.064198	1	913	3505	19840	244.06420	0.0123	81.290	28%
Americium	Am	95	148	243	243.061373	1	1449	2880	13700	243.06137	0.0177	56.364	19%
Curium	Cm	96	151	247	247.070347	1	1620	-	13500	247.07035	0.0183	54.640	19%

Key:
Non-metal
Semi-metal
Metal
Lanthanide
Actinide
Gas
Liquid
Solid
Synthetic

Melting Point : @ 26 atm, Sublimation

Mass Density @ 300 K (gases @ 273.15 K) and 1 atm.

Source of Data: Cross-checked from various sources

Avogadro's principle: All gases @ STP: 22.414 L = 1 mol

Gold's principle: All matter $R_v = 1 \text{ mol}$

Substance (samples)

Ice (273 K)	H ₂ O	10	8	18	18.015287	1	273	-	917	18.01529	0.0196	50.901	17%
Water (277 K)	H ₂ O	10	8	18	18.015287	1	273	373	999.973	18.01529	0.0180	55.507	19%
Steam (373.15 K)	H ₂ O	10	8	18	18.015287	1	273	373	0.60	18.01529	30.025	0.033	0.011%
Air (4N ₂ + O ₂) (Approx.)					14.405000	2			1.29	28.81000	22.333	0.045	0.015%
Ozone (O ₃)		8	8	16	15.999405	3	80.7	161.3	2.144	47.99821	22.387	0.045	0.015%
Methane	CH ₄	10	6	16	16.042500	1	90.7	111.66	0.716	16.04250	22.406	0.045	0.015%
Ammonia	NH ₃	10	7	17	17.030566	1	195.42	239.8	0.77	17.03057	22.118	0.045	0.015%
Carbon dioxide	CO ₂	22	22	44	44.009546	1	195	-	1.98	44.00955	22.227	0.045	0.015%
Dimethyl ether (298 K)	CH ₃ OCH ₃	26	20	46	46.068518	1	156.85	307.8	72.72	46.06852	0.6335	1.579	0.539%

Liquid Gas (samples)

Oxygen (90 K) (O ₂)	O	8	8	16	15.999405	2	54.8	90.188	1140	31.99881	0.0281	35.626	12%
Methane (109 K)	CH ₄	10	6	16	16.042500	1	90.7	111.66	464.54	16.04250	0.0345	28.957	10%

