

# Precise Molality and Utilization with Real Gases

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## **Abstract**

The objective of this paper is twofold. Firstly, in extending the current thermodynamical properties of mass, temperature, volume and pressure to include the substantiation of mass density as a thermodynamic property. It is shown that volumetric mass density is not just a straightforward ratio of mass and volume but a representation of a true ratio when heterogeneous states of matter are in equilibrium of state. It is further shown, for a given gas of constant mass, the product of density and temperature remain constant irrespective of induced change in volume and/or pressure. The second objective is the introduction of a methodological approach in establishing precise molality by refinement of the Avogadro principle, thereby allowing precise computation in the behaviour of real gases.

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## **1. Introduction**

The subject of gases is extensive and is attributed to many great contributors of the past. This paper deals only with new aspects regarding gases and the inherent imprecisions in the computability of gases where, in relation to this, rectification is long overdue and paramount in future education on the subject.

The Boltzmann constant, named after Ludwig Boltzmann, is a physical constant statistically relating the mean kinetic energy of particles in gases with the temperature of gases [1]. It has culminated in the universal use of the gas constant. Inevitably, it has led to the development of the ideal gas law in 1834 as an approximation in the computation of gases.

Named after Amedeo Avogadro, Avogadro's principle of 1811, or sometimes referred to as Avogadro's law, states: *equal volumes of all gases, at the same temperature and pressure, have the same number of molecules* [2]. It will be demonstrated that the stated term 'equal volumes' is imprecise when comparing different gases, and rectification will further remove inconsistencies in the computation of gases.

## 2. The Properties of Gases

The properties of mass and volume are dependent on the amount of a substance and are classified as *extensive* properties. In distinction, the properties of mass density, temperature and pressure are independent of the amount of a substance and are classified as *intensive* properties.

As exemplified by a quantity of gas divided into sub-portions where different masses and volumes, in each sub-portion, retain proportionality to the original; then the outcome of mass density, temperature and any pressure within the gas remain unaltered in each sub-portion.

In considering that the intensive property of mass density is a ratio of the extensive properties, each property can be ordered by dominancy; based on the priority of their interdependency on each other. The order is: mass, temperature, volume, pressure and mass density. The permutation of interdependencies between intensive properties will be highlighted during the course of this paper.

### 2.1 Pressure

It is important to distinguish between two fundamental forms of pressure: accumulative and induced pressure. Accumulative pressure is the result of gravitational influence and which attributes to matter. It is consequential to atmospheric pressure and increasingly accumulating within matter where it can ultimately culminate, at the quantum scale, in the degeneracy pressure of electrons (or neutrons in the case of neutron stars). *Mass density is dependent upon accumulative pressure*. Induced pressure, however, is the result of thermodynamical or induced physical effects and, is later shown, does not attribute to matter but remains as a thermodynamic attribution.

### 2.2 Intermolecular Forces

In deriving the gas equation from the kinetic theory, was based on a fundamental assumption that the molecules exert no forces on each other. It is known, however, that large intermolecular forces of attraction exist in liquids and solids (cohesive forces) and, therefore, the same forces must exist in the gaseous states, but are much feebler in strength due to the greater average distance between molecules [3]. This then infers intermolecular forces decrease in strength with increasing molecular separation. Molecular kinetic motion/energy is dependent upon temperature. Also, *pressure is dependent upon temperature*. There would at first appear to be a correlation between molecular kinetic motion/energy and pressure and, in fact, this is as presented in the kinetic theory; the notion that pressure results from the accumulative kinetic energy of atoms or molecules striking the internal container's surface/s [4].

It is forwarded from alternative reasoning both accumulative and induced pressure result from *static* intermolecular repulsive forces, in that strength is unaffected by temperature and, by the same reasoning, it is the strength of intermolecular attractive/cohesive forces which is affected and, therefore, dependent upon temperature: the strength of this *dynamic* force being inversely proportional to molecular kinetic motion/energy. Equivalence in interactions between these two opposing forces will produce equilibrium of state within a closed system, equating to symmetry between the two forces, whereby, any loss of equilibrium will correspond to differential in force strengths arising from spontaneous symmetry-breaking. Asymmetry in these force strengths will be the cause of expansion or contraction in matter, until equilibrium of state is re-established, and any restricted asymmetry in the gaseous states will be the cause of induced pressure.

### 2.3 Mass, Volume and Density

Mass major components comprises of volume and density. The properties of density  $\rho$ , volume  $V$  and induced pressure  $p$  of a gas are considered in the following equation where mass  $m$  and atmospheric pressure  $p_a$  remain constant:

$$\frac{m p_a}{V p} = \rho \quad (2.31)$$

Eqn. 2.31 forms Proposition 1. In addition to the well understood changes in pressure and volume, variability in mass density is accounted for in the following theorem:

**Theorem 1:** *In relation to atmospheric pressure, mass density of a given gas is proportional to the mass of the gas and inversely proportional to the product of volume and induced pressure.*

**Lemma 1:** In the case of unsuppressed volume of a gas, change in temperature will cause change in volume but pressure remains constant to atmospheric pressure; in conformity with Charles' law of 1787 [5]. Resultant change in mass density of Eqn. 2.31 is consistent with theorem 1.

**Lemma 2:** In the case of fixed volume of a gas, change in temperature will cause change in pressure but volume remains constant; in conformity with Gay-Lussac's (Amontons') Law of 1808 [6-7]. Resultant change in mass density of Eqn. 2.31 is consistent with theorem 1.

**Lemma 3:** In the case of compressed volume of a gas, in which initial constant temperature is reattained, will cause change in pressure and volume; in conformity with Boyle–Mariotte law of 1654 [8]. Resultant mass density of Eqn. 2.31 remains constant and is consistent with theorem 1.

	Mass of Gas	Initial Temperature	<i>Specimen</i> Temperature	Initial Volume	Resultant/ <i>Specimen</i> Volume	Initial Mass Density	Resultant Mass Density	Resultant Pressure	Constant $\rho' T'$
Volume	g	K	K	L	L	g/L	g/L	atm	
<b>Lemma 1: Unsuppressed</b>	<b>222</b>	273	<b>573</b>	22.816	<b>47.889</b>	9.73	<b>4.636</b>	<b>1</b>	<b>2656.290</b>
<b>Lemma 2: Fixed</b>	<b>222</b>	273	<b>573</b>	22.816	<b>22.816</b>	9.73	<b>4.636</b>	<b>2.099</b>	<b>2656.290</b>
<b>Lemma 3: Compressed</b>	<b>222</b>	273	<b>273</b>	22.816	<b>15</b>	9.73	<b>9.73</b>	<b>1.521</b>	<b>2656.290</b>

Table 1: Mass Density of a Gas <sup>1</sup>

From the above lemmas it can be obtain from deduction that change in density only occurs when there is change in temperature:

$$\frac{1}{V_i} \left( \frac{\partial V}{\partial T} \right)_p = \frac{1}{p_i} \left( \frac{\partial p}{\partial T} \right)_V = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_m \quad (2.32)$$

Lemma 1      Lemma 2      Lemmas 1 & 2

The deduction is further substantiated by the fact that for a given gas of constant mass the product of density and temperature remain constant. *Mass density is dependent upon temperature:*

$$\rho_1 T_1 = \rho_2 T_2 \quad (2.33)$$

<sup>1</sup> See criterion in 3 i.e.  $V_i = m/\rho_i$ . Calculated results are displayed rounded to three decimal places.

Theorem 1 has been validated and is furthered by the fact that the product of volume and induced pressure is consistent with the unsuppressed volume of a gas. Validation of theorem 1 concludes in the following classical laws:

Law 1: *In relation to atmospheric pressure, mass density of a given gas remains proportional to the mass and inversely proportional to the unsuppressed volume of the gas irrespective of change in volume and/or induced pressure.*

Law 2: *For a given gas of constant mass, mass density is inversely proportional to change in absolute temperature and in which the product of density and temperature remain constant.*

It has been established that volumetric mass density remains consistent with unsuppressed volume, which is a manifestation of equilibrium of state. By deduction, whereas accumulative pressure attributes to mass density, induced pressure does not affect mass density in any way. This will be a significant paradigm shift in our understanding of mass density; in that it is not just a straightforward ratio of mass and volume but is a representation of a true ratio when substances are in equilibrium of state and this will be applicable to all heterogeneous states of matter. The significance is further emphasized by the implication: as the alternative reasoning, presented in the topic of subsection 2.2, is essential to explaining the cause.

### 3. Precise Molality

In this context, and throughout this paper, molality is used as a generic term relating to the mole. The unit of a mole (shorten from reference to molecules), as derived by Avogadro, is a comparative or relative unit applicable to all matter states. For the purpose of clarity and consistency, the word ‘relative’ is used in molar components, and the upper case *R* used in the notation of variables that are pertaining to the mole; whereby these variables are then distinguishable concerning content.

A criterion exists in that to achieve greater accuracy in computation it will entail the use of a higher degree of precision.

#### 3.1 Atomic Mass Units

Atomic mass units are based on the average naturally occurring isotopes found in each atomic element. The masses used to compute the molecular mass are found in a table of isotopic masses and are not found in a typical periodic table. As convention, for each element and substance the notations used are *u* = atomic mass units and *N* = molecular element factor, which is applicable to elements only as the number of atoms combined to form a molecular element, else in all other instances *N* = 1. For substances, the atomic mass units are obtained as illustrated:

Substance		Protons	Neutrons	Nucleon Number	Initial Atomic Mass Units <i>u</i>	Resultant Atomic Mass Units <i>u</i>
Dimethyl ether CH <sub>3</sub> OCH <sub>3</sub>	C	6	6	12	12.011 × 1	12.011
	H <sub>3</sub>	3	0	3	1.00794 × 3	3.02382
	O	8	8	16	15.9994 × 1	15.9994
	C	6	6	12	12.011 × 1	12.011
	H <sub>3</sub>	3	0	3	1.00794 × 3	3.02382
<b>Dimethyl ether:</b>		<b>26</b>	<b>20</b>	<b>46</b>		<b>46.06904</b>

Illustration 1: Atomic Mass Units of a Substance

### 3.2 Relative Aspects of Matter

In establishing the following relative matter components, units of grams and litres are applied universally. Constant mass of an individual element or substance is uniformly expressed by its molar ‘relative’ mass:

$$R_m = uNm_uN_A \quad \text{g mol}^{-1} \quad (3.21)$$

where  $m_u$  = atomic mass constant (in grams) and  $N_A$  = Avogadro constant. Molar ‘relative’ volume is proportional to molar relative mass and inversely proportional to mass density<sup>1</sup>:

$$R_v = \frac{R_m}{\rho} \quad \text{L mol}^{-1} \quad (3.22)$$

where  $\rho$  = mass density. Precise molality is demonstrated by the slight variance in molar relative volume for each gas and, consequentially, establishes a crucial difference between gases. Consistency between the different mass densities of individual elements and substances is achieved by conversion to a relative ratio, referred to as ‘Relative Matter Density’:

$$R_\rho = \frac{d \ln R_m}{dR_v} \quad (3.23a)$$

$$\rho = uNR_\rho \quad (3.23b)$$

$$uNR_\rho = \frac{uNm_uN_A}{R_v} \quad (3.23c)$$

As  $m_uN_A = 1$  gram, relative matter density is thereby reciprocal to molar relative volume and is consistent with Eqn. 3.23a:

$$R_\rho = \frac{1}{R_v} \quad (3.23d)$$

Alternatively, relative matter density can initially be derived from:

$$R_\rho = \frac{\rho}{uN} \quad \text{g L}^{-1} \text{u}^{-1} \quad (3.23e)$$

A gaseous system is a closed system defined by its volume. Application of the molar unit becomes apparent when the size of a homogeneous system is unknown, whereby it allows evaluating a sub-portion of the system in which the results obtained will be consistent to the whole system. Or in the case of large heterogeneous systems, it allows evaluating sub-portions of the system, for example, at different pressures (atmospheric or induced).

If the initial volume is known, then the number of moles of a system is straightforwardly derived from the initial volume being inversely proportional to the molar relative volume:

$$n = \frac{V_i}{R_v} \quad (3.24)$$

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<sup>1</sup> Different temperatures and atmospheric pressures affect mass density and, thereby, although not essential, it is by way of consistency for the relative matter components of different elements and substances to be initialized at STP and, in doing so, will emphasize their comparative ‘relative’ aspects.

A specific gas constant relevant for a given gas is proportional to the product of molar relative volume and atmospheric pressure (in any specified units) and inversely proportional to the initializing temperature:

$$R_s = \frac{R_v p_a}{T_i} \quad (3.25)$$

The specific gas constant, when substituted into the ideal gas law, converts it to a specific gas law and, from its formulation, is reconcilable with molar relative volume:

$$R_s T = R_v (T/T_i) p_a \quad (3.26a)$$

Thereby, this will give rise to a new equation of state for gases without the need for a gas constant:

$$pV = nR_v (T/T_i) p_a \quad (3.26b)$$

Law 3: *In relation to atmospheric pressure, the product of induced pressure and volume of a given gas is directly proportional to its molar relative volume in which is proportional to any change in absolute temperature.*

Elements (gases only)	Element	Protons (Element: Atomic Number)		Neutrons	Nucleon Number	Atomic Mass Units u	Molecular Element Factor N	Mass Density $\rho$ (g/L or kg/m <sup>3</sup> )	Molar Relative Mass $R_m$ (g mol <sup>-1</sup> )	Molar Relative Volume $R_v$ (L mol <sup>-1</sup> )	Relative Matter Density $R_p$ (g L <sup>-1</sup> u <sup>-1</sup> )	Specific Gas Constant $R_s$ (dm <sup>3</sup> atm K <sup>-1</sup> mol <sup>-1</sup> )
		Atomic Number	Neutrons									
Hydrogen	H <sub>2</sub>	1	0	1	1.00794	2	0.0899	2.01588	22.42358	0.04460	0.08209	
Helium	He	2	2	4	4.0026	1	0.1785	4.0026	22.42353	0.04460	0.08209	
Nitrogen	N <sub>2</sub>	7	7	14	14.0067	2	1.251	28.0134	22.39281	0.04466	0.08198	
Oxygen	O <sub>2</sub>	8	8	16	15.9994	2	1.429	31.9988	22.39244	0.04466	0.08198	
Fluorine	F <sub>2</sub>	9	10	19	18.9984	2	1.696	37.9968	22.40377	0.04464	0.08202	
Neon	Ne	10	10	20	20.1797	1	0.900	20.1797	22.42189	0.04460	0.08209	
Chlorine	Cl <sub>2</sub>	17	18	35	35.4527	2	3.214	70.9054	22.06142	0.04533	0.08077	
Argon	Ar	18	22	40	39.948	1	1.784	39.948	22.39238	0.04466	0.08198	
Krypton	Kr	36	48	84	83.8	1	3.75	83.8	22.34667	0.04475	0.08181	
Xenon	Xe	54	77	131	131.29	1	5.9	131.29	22.25254	0.04494	0.08147	
Radon	Rn	86	136	222	222	1	9.73	222	22.81603	0.04383	0.08353	
<b>Substances (sample gases)</b>												
Ozone	O <sub>3</sub>	8	8	16	15.9994	3	2.144	47.9982	22.38722	0.04467	0.08196	
Methane	CH <sub>4</sub>	10	6	16	16.04276	1	0.716	16.04276	22.40609	0.04463	0.08203	
Ammonia	NH <sub>3</sub>	10	7	17	17.03052	1	0.77	17.03052	22.11756	0.04521	0.08097	
Steam (373.15 K)	H <sub>2</sub> O	10	8	18	18.0153	1	0.60	18.0153	30.02550	0.03331	0.08046	
Carbon dioxide	CO <sub>2</sub>	22	22	44	44.0098	1	1.98	44.0098	22.22717	0.04499	0.08137	

Table 2: Relative Matter Components of Gases at STP

Table A in the appendix contains a list of relative matter component quantities for each element and sample substances at STP or where temperatures for non-gases are specified. It is ascertained from the information in this table that  $R_v < 1$  for all liquids and solids and  $R_v > 1$  for all gases. Furthermore, there will be a divergent state when  $R_v = 1$ ; as, within the bounds of a critical temperature, momentary occurring at the approximate mean point during a phase transition involving vaporization/condensation or sublimation/deposition. These  $R_v$  conformities are applicable for all individual elements and substances and will be irrespective of pressure.

This forwards Rule 1: *At or below a critical temperature and irrespective of pressure, the proximal phase transition between a liquid or sublimation of a solid to a gas or vice versa of an individual element or substance (or in their summing) occurs at the point when their molar relative volume transcends 1 litre.*

### 3.3 Utilization with Real Gases

Earth's Atmospheric Gases	Atomic Mass Units	Molecular Element Factor	Mass Density $\rho$ (g/L or kg/m <sup>3</sup> ) @ STP	Molar Relative Mass $R_m$ (g mol <sup>-1</sup> )	Molar Relative Volume $R_v$ (L mol <sup>-1</sup> )	% of Earth's Atmospheric Gases by Volume	Fractional Molar Relative Volume by %	Fractional Molar Relative Mass	Fractional Molar Relative Matter Density	Dalton's partial pressures @ 323 K (L atm)		
Nitrogen	N <sub>2</sub>	14.0067	2	1.251	28.013	22.39281	78.0825	17.4848625	0.780829858	21.8736991	0.0348697	20.6758580
Oxygen	O <sub>2</sub>	15.9994	2	1.429	31.999	22.39244	20.944	4.6898731	0.209437903	6.7017616	0.0093531	5.5457771
Argon	Ar	39.948	1	1.784	39.948	22.39238	0.932	0.2086970	0.009319879	0.3723105	0.0004162	0.2467842
Carbon dioxide	CO <sub>2</sub>	44.0098	1	1.98	44.01	22.22717	0.039	0.0086686	0.000387118	0.0170370	0.0000174	0.0102506
Neon	Ne	20.1797	1	0.900	20.18	22.42189	0.001818	0.0004076	0.000018204	0.0003673	0.0000008	0.0004820
Helium	He	4.0026	1	0.1785	4.0026	22.42353	0.000524	0.0001175	0.000005247	0.0000210	0.0000002	0.0001389
Methane	CH <sub>4</sub>	16.0428	1	0.716	16.043	22.40609	0.000179	0.0000401	0.000001791	0.0000401	0.0000001	0.0000474
<b>Key: Atomic Element, Molecular Substance</b>				<b>Sum Total:</b>		<b>100</b>	<b>22.392666</b>	<b>1</b>	<b>28.965237</b>	<b>0.0446575</b>	<b>26.479338</b>	<b>L atm</b>
<b>Source: Wikipedia - Atmosphere of Earth</b>				<b>Specific Gas Constant:</b>		<b>0.0819794</b>	<b>Mass Density:</b>	<b>1.2935147</b>	<b>1.1825005</b>	<b>atm</b>		

Table 3: Composition of Earth's Atmosphere (dry air)

In using the Earth's atmosphere as an example it can be demonstrated that any gaseous mixture can be treated as a specific gas when the cumulative relative matter components are established by summing the component quantities for each individual gaseous elements and substances. (If the proportions are by percentage of mass, then the volume and mass aspects are interchanged.) For dry air at the Earth's surface where:

$$\begin{aligned} \text{Pressure:} & \quad p_a = 1 & \text{atm} \\ \text{Temperature:} & \quad T_i = 273.15 & \text{K} \end{aligned}$$

The relative matter components are:

$$\begin{aligned} \text{Molar relative mass:} & \quad R_m = 28.965237 & \text{g mol}^{-1} \\ \text{Molar relative volume:} & \quad R_v = 22.392666 & \text{L mol}^{-1} \\ \text{Relative matter density:} & \quad R_\rho = 0.0446575 & \text{g L}^{-1} \text{u}^{-1} \\ \text{Specific gas constant:} & \quad R_s = 0.0819794 & \text{dm}^3 \text{atm K}^{-1} \text{mol}^{-1} \\ \text{Mass density:} & \quad \rho = 1.2935147 & \text{g/L or kg/m}^3 \end{aligned}$$

In establishing the values for relative matter components of a gaseous mixture, aspects like pressure are straightforwardly calculated without the need in resorting to the application of Dalton's 1793 law of partial pressures [9]. In the example produced in table 3, the concluding pressure established by applying Dalton's partial pressures within a fixed volume of 1 mole and at 323 K can also be obtained by use of the specific gas constant:

$$\frac{R_s T}{R_v} = p \quad (3.31)$$

$$(0.0819794 \text{ dm}^3 \text{ atm K}^{-1} \text{ mol}^{-1} \times 323 \text{ K}) / 22.392666 \text{ L} = 1.1825005 \text{ atm}$$

Or in the application of the new specific equation of state for a given gas, as defined by Law 3, where in this instance of fixed volume,  $V = nR_v$  and therefore will cancel out leaving the differential in temperature that ascertains the change in pressure; as supported by Gay-Lussac's (Amontons') Law [6-7]:

$$\frac{nR_v T p_a}{V T_i} = p \quad (3.32)$$

$$(323 \text{ K} / 273.15 \text{ K}) \times 1 \text{ atm} = 1.1825005 \text{ atm}$$

### 3.3.1 Critical Point of Gases

Element (gases)	Data						Proposition 1 (Theorem 1)				Law 2	
	Atomic Mass Units u	Molecular Element Factor N	Mass Density $\rho$ @ $T_i$ (g/L or kg/m <sup>3</sup> )	Critical Pressure $P_c$ (atm)	Critical Temperature $T_c$ K	Molar Relative Mass $R_m$ (g/mol)	Molar Relative Volume $R'_v$ (L/mol)	Mass Density $\rho$ @ $T_c$ (g/L or kg/m <sup>3</sup> )	Reaffirms Critical Pressure $P_c$ (atm)	Constant $\rho_i T_i = \rho_c T_c$		
Hydrogen	H <sub>2</sub>	1.00794	2	0.0899	12.80	33.2	2.01588	0.21293	0.73964	12.80	24.55619	24.55619
Helium	He	4.00260	1	0.1785	2.24	5.19	4.0026	0.19021	9.39447	2.24	48.75728	48.75728
Nitrogen	N <sub>2</sub>	14.00670	2	1.251	33.50	126.2	28.0134	0.30883	2.70769	33.50	341.7107	341.7107
Oxygen	O <sub>2</sub>	15.99940	2	1.429	49.80	154.6	31.9988	0.25450	2.52478	49.80	390.3314	390.3314
Fluorine	F <sub>2</sub>	18.99840	2	1.696	51.50	144.3	37.9968	0.22982	3.21041	51.50	463.2624	463.2624
Neon	Ne	20.17970	1	0.900	27.20	44.4	20.1797	0.13399	5.53682	27.20	245.835	245.835
Chlorine	Cl <sub>2</sub>	35.45270	2	3.214	76.00	416.9	70.9054	0.44305	2.10579	76.00	877.9041	877.9041
Argon	Ar	39.94800	1	1.784	48.10	150.8	39.948	0.25701	3.23143	48.10	487.2996	487.2996
Krypton	Kr	83.80000	1	3.75	54.30	209.3	83.8	0.31534	4.89399	54.30	1024.313	1024.313
Xenon	Xe	131.29000	1	5.9	57.60	289.8	131.29	0.40988	5.56102	57.60	1611.585	1611.585
Radon	Rn	222.00000	1	9.73	61.98	377	222	0.50809	7.04973	61.98	2657.75	2657.75
<b>Substance (sample gases)</b>												
Methane	CH <sub>4</sub>	16.04276	1	0.716	45.79	190.8	16.04276	0.34180	1.02503	45.79	195.5754	195.5754
Ammonia	NH <sub>3</sub>	17.03052	1	0.77	111.30	405.5	17.03052	0.29501	0.51868	111.30	210.3255	210.3255
Steam (373.15 K)	H <sub>2</sub> O	18.01530	1	0.60	217.70	647.096	18.0153	0.23918	0.34599	217.70	223.89	223.89
Carbon dioxide	CO <sub>2</sub>	44.00980	1	1.98	72.80	304.19	44.0098	0.34001	1.77796	72.80	540.837	540.837

Table 4: Critical Point of Gases

Data for the critical point of gases is used to further substantiate Proposition 1 for Theorem 1: *In relation to atmospheric pressure, mass density of a given gas is proportional to the mass of the gas and inversely proportional to the product of volume and induced pressure.*

Constant mass of a given gas is as expressed by Eqn. 3.21. Molar relative volume, initially established at STP in table 2 where  $T_i = 273.15$  K (Steam:  $T_i = 373.15$  K) and  $p_a = 1$  atm, is updated to include critical temperature and critical pressure:

$$R'_v = R_v[(T_c/T_i)(p_a/p_c)] \quad (3.41)$$

From proposition 1:

$$\rho = \frac{R_m p_a}{R'_v p_c} \quad (3.42)$$

Equivalent mass density is derived from relative matter density, updated from table 2, using critical temperature only:

$$\rho = uNR_\rho(T_i/T_c) \quad (3.43)$$

(By rearrangement of proposition 1:  $p_c = (R_m p_a)/(R'_v \rho)$ .) Eqn. 3.43 reaffirms that induced pressure does not attribute to relative matter density, and, consequentially, mass density, and this will further substantiate Law 2: *For a given gas of constant mass, mass density is inversely proportional to change in absolute temperature and in which the product of density and temperature remain constant.*

Table 4 shows that molar relative volume has transcended the 1 litre threshold, as stated in Rule 1, at a transitional critical point, as in the above examples, where the gas transforms into a liquid (in compliance with critical temperature:  $R_v < 1$  for liquids and solids and  $R_v > 1$  for gases).



### 3.4 Equilibrium of State

The only change in pressure that attributes to relative matter density, and more predominately in the gaseous state, is the effect of accumulative atmospheric pressure. Thereby, the essential attributions that affect both molar relative volume and relative matter density of a specific gas are as follows:

$$R'_v = R_v[(T/T_i)(p_a/p)(p_a/p'_a)] \quad (3.51)$$

$$R'_\rho = R_\rho[(T_i/T)(p'_a/p_a)] \quad (3.52)$$

where  $p_a$  = atmospheric pressure and  $p$  = induced pressure. In all heterogeneous states of matter equilibrium of state is identifiable when the product  $R'_v R'_\rho = 1$ . In the gaseous state loss of equilibrium, due to an induced physical or thermodynamical effect, correlates with the resulting induced pressure; as the product of  $R'_v R'_\rho$  being inversely proportional to accumulative atmospheric pressure:

$$p = \frac{p_a}{R'_v R'_\rho} \quad (3.53)$$

## 4. Conclusion

From the established proportionality of temperature and mass density and from alternative reasoning relating to intermolecular forces it can be further deduced, conjecturally, that, within all heterogeneous states of matter, the *dynamic* intermolecular attractive/cohesive force will be responsible in determining mass density and in equipollence the *static* or consistency of the intermolecular repulsive force will be responsible in determining volume. Furthermore, it has been demonstrated that induced pressure does not attribute to matter, therefore, the overall conclusion is that induced pressure is not a direct consequence of kinetic energy (see lemma 3) but constitutes to potential energy of a gaseous system in which direct correlations between induced pressure, potential energy and loss of equilibrium of state must ultimately arise from asymmetry in opposing intermolecular force strengths.

## Applied constants

<http://physics.nist.gov/cuu/Constants/index.html>

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# Appendix

Table A: Relative Matter Components of Elements and Substances

Element	Periodic Table Symbol	Protons (Element: Atomic Number)	Neutrons	Nucleon Number	Atomic Mass Units u	Molecular Element Factor N	Melting Point (K)	Boiling Point (K)	Mass Density $\rho$ (g/L or kg/m <sup>3</sup> )	Molar Relative Mass $R_m$ (g mol <sup>-1</sup> )	Molar Relative Volume $R_v$ (L mol <sup>-1</sup> )	Relative Matter Density $R_p$ (g L <sup>-1</sup> u <sup>-1</sup> )	$R_p$ % of Diamond
Hydrogen (H <sub>2</sub> )	H	1	0	1	1.00794	2	13.81	20.28	0.0899	2.01588	22.424	0.045	0.015%
Helium	He	2	2	4	4.00260	1	0.95	4.216	0.1785	4.00260	22.424	0.045	0.015%
Lithium	Li	3	4	7	6.94100	1	453.7	1615	530	6.94100	0.0131	76.358	26%
Beryllium	Be	4	5	9	9.01218	1	1560	3243	1850	9.01218	0.0049	205.278	70%
Boron	B	5	6	11	10.81100	1	2365	4275	2340	10.81100	0.0046	216.446	74%
Carbon	C	6	6	12	12.01100	1	3825	5100	2260	12.01100	0.0053	188.161	64%
Diamond		6	6	12	12.01100	1	3825	5100	3520	12.01100	0.0034	293.065	100%
Graphene		6	6	12	12.01100	1	3825	5100	4120	12.01100	0.0029	343.019	117%
Nitrogen (N <sub>2</sub> )	N	7	7	14	14.00670	2	63.15	77.344	1.251	28.01340	22.393	0.045	0.015%
Oxygen (O <sub>2</sub> )	O	8	8	16	15.99940	2	54.8	90.188	1.429	31.99880	22.392	0.045	0.015%
Fluorine (F <sub>2</sub> )	F	9	10	19	18.99840	2	53.55	85	1.696	37.99680	22.404	0.045	0.015%
Neon	Ne	10	10	20	20.17970	1	24.55	27.1	0.900	20.17970	22.422	0.045	0.015%
Sodium	Na	11	12	23	22.98977	1	371	1156	970	22.98977	0.0237	42.193	14%
Magnesium	Mg	12	12	24	24.30500	1	922	1380	1740	24.30500	0.0140	71.590	24%
Aluminium	Al	13	14	27	26.98154	1	933.5	2740	2700	26.98154	0.0100	100.068	34%
Silicon	Si	14	14	28	28.08550	1	1683	2630	2330	28.08550	0.0121	82.961	28%
Phosphorus (White)	P	15	16	31	30.97376	1	317.3	553	1820	30.97376	0.0170	58.759	20%
Phosphorus (Red)	P	15	16	31	30.97376	1	317.3	553	2200	30.97376	0.0141	71.028	24%
Phosphorus (Black)	P	15	16	31	30.97376	1	317.3	553	2700	30.97376	0.0115	87.171	30%
Sulfur	S	16	16	32	32.06600	1	392.2	717.82	2070	32.06600	0.0155	64.554	22%
Chlorine (Cl <sub>2</sub> )	Cl	17	18	35	35.45270	2	172.2	239.18	3.214	70.90540	22.061	0.045	0.015%
Argon	Ar	18	22	40	39.94800	1	83.95	87.45	1.784	39.94800	22.392	0.045	0.015%
Potassium	K	19	20	39	39.09830	1	336.8	1033	860	39.09830	0.0455	21.996	8%
Calcium	Ca	20	20	40	40.07800	1	1112	1757	1550	40.07800	0.0259	38.675	13%
Scandium	Sc	21	24	45	44.95590	1	1814	3109	2990	44.95590	0.0150	66.510	23%
Titanium	Ti	22	26	48	47.87000	1	1935	3560	4540	47.87000	0.0105	94.840	32%
Vanadium	V	23	28	51	50.94150	1	2163	3650	6110	50.94150	0.0083	119.942	41%
Chromium	Cr	24	28	52	51.99600	1	2130	2945	7190	51.99600	0.0072	138.280	47%
Manganese	Mn	25	30	55	54.93800	1	1518	2235	7440	54.93800	0.0074	135.425	46%
Iron	Fe	26	30	56	55.84500	1	1808	3023	7874	55.84500	0.0071	140.997	48%
Cobalt	Co	27	32	59	58.93320	1	1768	3143	8900	58.93320	0.0066	151.018	52%
Nickel	Ni	28	31	59	58.69340	1	1726	3005	8900	58.69340	0.0066	151.635	52%
Copper	Cu	29	35	64	63.54600	1	1356.6	2840	8960	63.54600	0.0071	141.000	48%
Zinc	Zn	30	35	65	65.39000	1	692.73	1180	7130	65.39000	0.0092	109.038	37%
Gallium	Ga	31	39	70	69.72300	1	302.92	2478	5910	69.72300	0.0118	84.764	29%
Germanium	Ge	32	41	73	72.61000	1	1211.5	3107	5320	72.61000	0.0136	73.268	25%
Arsenic	As	33	42	75	74.92160	1	889	-	5780	74.92160	0.0130	77.147	26%
$\alpha$ -Arsenic	As	33	42	75	74.92160	1	889	-	2000	74.92160	0.0375	26.695	9%
Selenium	Se	34	45	79	78.96000	1	494	958	4790	78.96000	0.0165	60.664	21%
Bromine (Br <sub>2</sub> )	Br	35	45	80	79.90400	2	265.95	331.85	3120	159.80800	0.0512	19.523	7%
Krypton	Kr	36	48	84	83.80000	1	116	120.85	3.75	83.80000	22.347	0.045	0.015%
Rubidium	Rb	37	48	85	85.46780	1	312.63	961	1532	85.46780	0.0558	17.925	6%
Strontium	Sr	38	50	88	87.62000	1	1042	1655	2540	87.62000	0.0345	28.989	10%
Yttrium	Y	39	50	89	88.90590	1	1795	3611	4470	88.90590	0.0199	50.278	17%
Zirconium	Zr	40	51	91	91.22400	1	2128	4682	6510	91.22400	0.0140	71.363	24%
Niobium	Nb	41	52	93	92.90640	1	2742	5015	8570	92.90640	0.0108	92.243	31%
Molybdenum	Mo	42	54	96	95.94000	1	2896	4912	10220	95.94000	0.0094	106.525	36%
Technetium	Tc	43	55	98	98.00000	1	2477	4538	11500	98.00000	0.0085	117.347	40%
Ruthenium	Ru	44	57	101	101.07000	1	2610	4425	12370	101.07000	0.0082	122.390	42%
Rhodium	Rh	45	58	103	102.90550	1	2236	3970	12410	102.90550	0.0083	120.596	41%
Palladium	Pd	46	60	106	106.42000	1	1825	3240	12000	106.42000	0.0089	112.761	38%
Silver	Ag	47	61	108	107.86800	1	1235.08	2436	10500	107.86800	0.0103	97.341	33%
Cadmium	Cd	48	64	112	112.41000	1	594.26	1040	8650	112.41000	0.0130	76.950	26%
Indium	In	49	66	115	114.82000	1	429.78	2350	7310	114.82000	0.0157	63.665	22%
Tin (White)	Sn	50	69	119	118.71000	1	505.12	2876	7310	118.71000	0.0162	61.579	21%
Tin (Grey)	Sn	50	69	119	118.71000	1	505.12	2876	5800	118.71000	0.0205	48.859	17%
Antimony	Sb	51	71	122	121.76000	1	903.91	1860	6690	121.76000	0.0182	54.944	19%
Tellurium	Te	52	76	128	127.60000	1	722.72	1261	6240	127.60000	0.0204	48.903	17%
Iodine	I	53	74	127	126.90450	1	386.7	457.5	4930	126.90450	0.0257	38.848	13%
Xenon	Xe	54	77	131	131.29000	1	161.39	165.1	5.9	131.29000	22.253	0.045	0.015%
Caesium	Cs	55	78	133	132.90540	1	301.54	944	1870	132.90540	0.0711	14.070	5%
Barium	Ba	56	81	137	137.33000	1	1002	2078	3590	137.33000	0.0383	26.141	9%

Element	Periodic Table Symbol	Protons (Element Atomic Number)	Neutrons	Nucleon Number	Atomic Mass Units u	Molecular Element Factor N	Melting Point (K)	Boiling Point (K)	Mass Density $\rho$ (g/L or kg/m <sup>3</sup> )	Molar Relative Mass $R_m$ (g mol <sup>-1</sup> )	Molar Relative Volume $R_v$ (L mol <sup>-1</sup> )	Relative Matter Density $R_p$ (g L <sup>-1</sup> u <sup>-1</sup> )	$R_p$ % of Diamond
Lanthanum	La	57	82	139	138.90550	1	1191	3737	6150	138.90550	0.0226	44.275	15%
Cerium	Ce	58	82	140	140.12000	1	1071	3715	6770	140.12000	0.0207	48.316	16%
Praseodymium	Pr	59	82	141	140.90770	1	1204	3785	6770	140.90770	0.0208	48.046	16%
Neodymium	Nd	60	84	144	144.24000	1	1294	3347	7010	144.24000	0.0206	48.600	17%
Promethium	Pm	61	84	145	145.00000	1	1315	3273	7220	145.00000	0.0201	49.793	17%
Samarium	Sm	62	88	150	150.36000	1	1347	2067	7520	150.36000	0.0200	50.013	17%
Europium	Eu	63	89	152	151.96400	1	1095	1800	5240	151.96400	0.0290	34.482	12%
Gadolinium	Gd	64	93	157	157.25000	1	1585	3545	7900	157.25000	0.0199	50.238	17%
Terbium	Tb	65	94	159	158.92530	1	1629	3500	8230	158.92530	0.0193	51.785	18%
Dysprosium	Dy	66	97	163	162.50000	1	1685	2840	8550	162.50000	0.0190	52.615	18%
Holmium	Ho	67	98	165	164.93030	1	1747	2968	8800	164.93030	0.0187	53.356	18%
Erbium	Er	68	99	167	167.26000	1	1802	3140	9070	167.26000	0.0184	54.227	19%
Thulium	Tm	69	100	169	168.93420	1	1818	2223	9320	168.93420	0.0181	55.169	19%
Ytterbium	Yb	70	103	173	173.04000	1	1092	1469	6970	173.04000	0.0248	40.280	14%
Lutetium	Lu	71	104	175	174.96700	1	1936	3668	9840	174.96700	0.0178	56.239	19%
Hafnium	Hf	72	106	178	178.49000	1	2504	4875	13310	178.49000	0.0134	74.570	25%
Tantalum	Ta	73	108	181	180.94790	1	3293	5730	16650	180.94790	0.0109	92.015	31%
Tungsten	W	74	110	184	183.84000	1	3695	5825	19300	183.84000	0.0095	104.983	36%
Rhenium	Re	75	111	186	186.20700	1	3455	5870	21000	186.20700	0.0089	112.778	38%
Osmium	Os	76	114	190	190.23000	1	3300	5300	22600	190.23000	0.0084	118.804	41%
Iridium	Ir	77	115	192	192.22000	1	2720	4700	22600	192.22000	0.0085	117.574	40%
Platinum	Pt	78	117	195	195.08000	1	2042.1	4100	21450	195.08000	0.0091	109.955	38%
Gold	Au	79	118	197	196.96650	1	1337.58	3130	19300	196.96650	0.0102	97.986	33%
Mercury	Hg	80	121	201	200.59000	1	284.31	629.88	13550	200.59000	0.0148	67.551	23%
Thallium	Tl	81	123	204	204.38300	1	577	1746	11850	204.38300	0.0172	57.979	20%
Lead	Pb	82	125	207	207.20000	1	600.65	2023	11350	207.20000	0.0183	54.778	19%
Bismuth	Bi	83	126	209	208.98040	1	544.59	1837	9750	208.98040	0.0214	46.655	16%
Polonium	Po	84	125	209	209.00000	1	527	1237	9300	209.00000	0.0225	44.498	15%
Astatine	At	85	125	210	210.00000	1	575	610	7000	210.00000	0.0300	33.333	11%
Radon	Rn	86	136	222	222.00000	1	202	211.4	9.73	222.00000	22.816	0.044	0.015%
Francium	Fr	87	136	223	223.00000	1	300	950	1870	223.00000	0.1193	8.386	3%
Radium	Ra	88	138	226	226.00000	1	973	1413	5000	226.00000	0.0452	22.124	8%
Actinium	Ac	89	138	227	227.00000	1	1324	3470	10070	227.00000	0.0225	44.361	15%
Thorium	Th	90	142	232	232.03810	1	2028	5060	11720	232.03810	0.0198	50.509	17%
Protactinium	Pa	91	140	231	231.03590	1	1845	4300	15400	231.03590	0.0150	66.656	23%
Uranium	U	92	146	238	238.02900	1	1408	4407	18950	238.02900	0.0126	79.612	27%
Neptunium	Np	93	144	237	237.00000	1	912	4175	20200	237.00000	0.0117	85.232	29%
Plutonium	Pu	94	150	244	244.00000	1	913	3505	19840	244.00000	0.0123	81.311	28%
Americium	Am	95	148	243	243.00000	1	1449	2880	13700	243.00000	0.0177	56.379	19%
Curium	Cm	96	151	247	247.00000	1	1620	-	13500	247.00000	0.0183	54.656	19%

<b>Key:</b>
<b>Non-metal</b>
<b>Semi-metal</b>
<b>Metal</b>
<b>Lanthanide</b>
<b>Actinide</b>
<b>Gas</b>
<b>Liquid</b>
<b>Solid</b>
<b>Synthetic</b>

Melting Point : @ 26 atm, Sublimation

Mass Density @ 300 K (gases @ 273.15 K) and 1 atm.

Source of Data: Cross-checked from various sources

Avogadro's principle: All gases @ STP 22.414 L = 1 mol

Gold's principle: All matter  $R_v = 1$  mol

### Substance (samples)

Ice (273 K)	H <sub>2</sub> O	10	8	18	18.01530	1	273	-	917	18.01530	0.0196	50.901	17%
Water (277 K)	H <sub>2</sub> O	10	8	18	18.01530	1	273	373	999.973	18.01530	0.0180	55.507	19%
Steam (373.15 K)	H <sub>2</sub> O	10	8	18	18.01530	1	273	373	0.60	18.01530	30.026	0.033	0.011%
Air (4N <sub>2</sub> + O <sub>2</sub> ) (Approx.)					14.45000	2			1.29	28.90000	22.403	0.045	0.015%
Ozone (O <sub>3</sub> )		8	8	16	15.99940	3			2.144	47.99820	22.387	0.045	0.015%
Methane	CH <sub>4</sub>	10	6	16	16.04276	1	90.7	161.3	0.716	16.04276	22.406	0.045	0.015%
Carbon dioxide	CO <sub>2</sub>	22	22	44	44.00980	1	195	-	1.98	44.00980	22.227	0.045	0.015%
Ammonia	NH <sub>3</sub>	10	7	17	17.03052	1	195.42	239.8	0.77	17.03052	22.118	0.045	0.015%
Dimethyl ether (298 K)	CH <sub>3</sub> OCH <sub>3</sub>	26	20	46	46.06904	1	156.85	307.8	72.72	46.06904	0.6335	1.579	0.539%

### Liquid Gas (samples)

Oxygen (90 K) (O <sub>2</sub> )	O	8	8	16	15.99940	2	54.8	90.188	1140	31.99880	0.0281	35.626	12%
Methane (109 K)	CH <sub>4</sub>	10	6	16	16.04276	1	90.7	111.66	464.54	16.04276	0.0345	28.956	10%

